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OPTIMIZATION PROBLEMS FOR PLATES OSCILLATING IN AN IDEAL FLUID

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The present paper deals with the oscillations of elastic plates in an ideal fluid. The optimizing problem of determining the thickness distribution for which the fundamental oscillation frequency is a maximum, is formulated. Necessary conditions for the extremum are derived. The relation between the fundamental frequency (a functional) and the parameters of the problem is investigated. The asymptotic behavior of the thickness and deflection distributions at the edges of the optimal plate is studied. An analytic solution of the optimization problem is given for thin, three-layer panels and it is shown that in this case the conditions of optimality are not only necessary, but also sufficient. The problem was

solved numerically in [1] for the case of solid panels.

1. Let us consider the problem of small oscillations of an elastic plate in an infinite mass of ideal fluid. We assume that the plate is hinged along a smooth contour Γ in the plane $z = 0$ of the rectangular xyz-coordinate system. Let us denote by $h = h(x, y)$ and $u = u(x, y, t)$, $Q = Q(x, y, t)$ the thickness distribution, plate deflection function and the fluid reaction on the plate, respectively. Then the equation of small plate oscillations and the boundary conditions can be written in the form

$$\rho_1 h \frac{\partial^2 u}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(D \frac{\partial^2 u}{\partial x^2}\right) + \nu \frac{\partial^2}{\partial x^2}\left(D \frac{\partial^2 u}{\partial y^2}\right) + \nu \frac{\partial^2}{\partial y^2}\left(D \frac{\partial^2 u}{\partial x^2}\right) + \tag{1.1}$$

$$\frac{\partial^2}{\partial y^2}\left(D \frac{\partial^2 u}{\partial y^2}\right) + 2(1-\nu)\frac{\partial^2}{\partial x \partial y}\left(D \frac{\partial^2 u}{\partial x \partial y}\right) = Q, \quad (x, y) \in G$$

$$u = 0, \quad D\left(\Delta u - \frac{1-\nu}{R}\frac{\partial u}{\partial n}\right) = 0, \quad (x, y) \in \Gamma$$

Here G denotes a region in the xy-plane bounded by the contour Γ, $D = D(h)$ is the bending stiffness of the plate, ν is the Poisson's ratio and ρ is the specific density of the plate material. Δ, n and R denote, respectively, the Laplace operator acting on the variables x and y, the normal and the radius of curvature of the contour Γ.

We assume that the motion of the fluid is irrotational. In this case we can describe it with the help of the velocity potential $\varphi = \varphi(x, y, z,$ wich satisfies the Laplace equation and the linearized boundary conditions

$$\Delta\varphi = 0, \quad (\partial\varphi/\partial z)_G{}^{\pm} = \partial u/\partial t, \quad (\varphi)_\infty = 0 \tag{1.2}$$

The first of the boundary conditions in (1.2) refers to the upper (plus sign) and the lower (minus sign) edge of the slit $z = 0$, $(x, y) \in G$. This condition is obtained by transferring the condition of zero fluid flow across the plate surface, to the xy-plane, under the assumption that the deflections u and thicknesses h are small and the motion of the fluid is inseparable from the motions of the plate.

The reaction Q exerted on the plate by the fluid is equal to the pressure difference between the lower and upper surface of the plate, i.e. $Q = p^- - p^+$. Using the Cauchy-Lagrange integral we can express the distribution of pressure p in terms of the velocity potential φ. Neglecting the second order terms we obtain the expression $p = p_\infty - \rho_2 \partial\varphi / \partial t$, where ρ_2 is the fluid density and p_∞ denotes the pressure at infinity. In this manner we obtain the following expression for the reaction Q:

$$Q = \rho_2 \left(\frac{\partial\varphi^+}{\partial t} - \frac{\partial\varphi^-}{\partial t}\right) \tag{1.3}$$

The closed boundary value problem (1.1) — (1.3) defines the functions $u(x, y, t)$ and $\varphi(x, y, z, t)$ completely, provided that certain initial conditions are given.

In what follows, we shall be considering free oscillations of the system, consequently we shall seek a solution of the problem (1.1) — (1.3) in the form

$$u = U(x, y)\exp(i\omega t), \quad \varphi = i\omega\Phi(x, y, z)\exp(i\omega t)$$

where ω is the free oscillations frequency. Let us write the relation between the bending stiffness of the plate D and the distribution of thickness h in the form $D(h) = \beta_m h^m$. When $m = 3$, the expression corresponds to the case of solid plates, and for

$m = 1$ to the three-layer plates (here h is the thickness of the outer, reinforcing plates).

Let us pass to the dimensionless variables and notation

$$x' = x / l, \quad y' = y / l, \quad z' = z / l, \quad U' = U / l, \quad h' = l^2 h / V$$
$$\Omega^2 = \omega^2 \rho_1 l^{2m+2} \beta_m^{-1} V^{1-m}, \quad \alpha = \rho_2 l^3 / \rho_1 V$$

Here V denotes the plate volume, l is the characteristic dimension of G, α is the dimensionless parameter of the problem. (Below the primes are omitted). This yields the following boundary eigenvalue problem:

$$L_1(h, U) \equiv \frac{\partial^2}{\partial x^2}\left(h^m \frac{\partial^2 U}{\partial x^2}\right) + \nu \frac{\partial^2}{\partial x^2}\left(h^m \frac{\partial^2 U}{\partial y^2}\right) + \nu \frac{\partial^2}{\partial y^2}\left(h^m \frac{\partial^2 U}{\partial x^2}\right) + \tag{1.4}$$
$$+ \frac{\partial^2}{\partial y^2}\left(h^m \frac{\partial^2 U}{\partial y^2}\right) + 2(1-\nu)\frac{\partial^2}{\partial x \partial y}\left(h^m \frac{\partial^2 U}{\partial x \partial y}\right) = \Omega^2 [hU + \alpha(\Phi^- - \Phi^+)]$$
$$z = 0, \quad (x, y) \in G$$

$$(U)_\Gamma = 0, \quad h^m \left(\Delta U - \frac{1-\nu}{R}\frac{\partial U}{\partial n}\right)_\Gamma = 0 \tag{1.5}$$

$$\Delta \Phi = 0, \quad \left(\frac{\partial \Phi}{\partial z}\right)_G^{\pm} = U, \quad (\Phi_\infty) = 0 \tag{1.6}$$

2. Since the problem for the potential Φ is linear in U and independent of h, the difference $\Phi^- - \Phi^+$ in the right-hand side of (1.4) can be written as $\Phi^- - \Phi^+ = L_2 U$, where L_2 is a linear operator which is positive and selfconjugate. In fact, let U_1 and U_2 be two arbitrary functions and let Φ_1 and Φ_2 be the solutions of the boundary value problem (1.6) for the cases $U = U_1$ and $U = U_2$. Then

$$(U_1, L_2 U_2) = \iint_G U_1 L_2 U_2 \, dx\, dy = \iint_G U_1 (\Phi_2^- - \Phi_2^+)\, dx\, dy =$$
$$\iint_G \left(\frac{\partial \Phi_1}{\partial z}\Phi_2^- - \frac{\partial \Phi_1}{\partial z}\Phi_2^+\right) dx\, dy = -\iint_G \frac{\partial \Phi_1}{\partial n}\Phi_2 \, d\sigma$$

In the last integral the integration is carried out over the upper, as well as the lower surface of the plate, and n denotes the outer normal to the plate surface. Transforming the integral in accordance with the Green formula, we obtain

$$(U_1, L_2 U_2) = -\oiint_G \frac{\partial \Phi_1}{\partial n}\Phi_2 \, d\sigma = -\oiint_G \Phi_1 \frac{\partial \Phi_2}{\partial n}\, d\sigma = \iint_G (\Phi_1^- - \Phi_1^+) \times$$
$$\frac{\partial \Phi_2}{\partial z}\, dx\, dy = \iint_G L_2 U_1 U_2 \, dx\, dy = (L_2 U_1, U_2)$$

Next, we shall show that $(U, L_2 U) > 0$ for all admissible U. In fact, for any admissible value of U we have

$$(U, L_2 U) = \iint_G U L_2 U dx\, dy = \iint_G \frac{\partial \Phi}{\partial z}(\Phi^- - \Phi^+)\, dx\, dy =$$
$$-\oiint_G \Phi \frac{\partial \Phi}{\partial n} d\sigma = \iiint (\operatorname{grad} \Phi)^2 \, dx\, dy\, dz > 0$$

where the integration in the last integral is carried out over the whole region occupied by the fluid.

The positiveness and selfconjugation properties of the operator L_1 are well known (see [2]).

Thus the eigenvalue problem (1.4) — (1.6) is selfconjugate and fully defined. This implies that the eigenvalues are real, and the Rayleigh variational principle can be used to find the smallest eigenvalue

$$\Omega_0^2(h) = \min_U J(h, U) \tag{2.1}$$

$$J(h, U) = I_1(h, U)[\alpha I_2(U) + I_3(h, U)]^{-1}$$

$$I_1 = \iint_G U L_1 U\,dx\,dy, \quad I_2 = \iint_G U L_2 U\,dx\,dy, \quad I_3 = \iint_G hU^2\,dx\,dy$$

The minimum in U is calculated on the set of all twice continuously differentiable functions $U(x, y)$ satisfying the first boundary condition of (1.5). The second boundary condition of (1.5) need not be satisfied a priori ; the function U minimizing the functional J will satisfy the second boundary condition of (1.5) automatically.

3. The Rayleigh relation $J(h, U)$ is a functional of $h(x, y)$ and $U(x, y)$, therefore the fundamental frequency Ω_0 determined from (2.1), depends on $h(x, y)$. Let us consider the set of continuous functions $h(x, y)$ satisfying the condition

$$\iint_G h\,dxdy = 1 \tag{3.1}$$

The isoperimetric condition (3.1) expresses the fact that the volume of the plate is constant. It is made equal to unity, by a suitable choice of the dimensionless variables.

Let us formulate the optimization problem. We require to find a function h satisfying condition (3.1) and maximizing the smallest eigenvalue Ω_0 (the fundamental frequency), i.e.

$$\Omega_{0*}^2 = \max_h \Omega_0^2(h) = \max_h \min_U J(h, U) \tag{3.2}$$

The problem (3.1), (3.2) has a single parameter $\alpha = \rho_2 l^3 / (\rho_1 V)$. When $\alpha = 0$, we have the problem of frequency optimization for a plate oscillating in vacuum.

Let us find the necessary conditions of optimality in the problem (3.1), (3.2). Writing the expressions for the first variation δJ under the condition (3.1) and assuming $\delta J = 0$, we obtain

$$mh^{m-1}\left\{\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right)^2 - 2(1-\nu)\left[\frac{\partial^2 U}{\partial x^2}\frac{\partial^2 U}{\partial y^2} - \left(\frac{\partial^2 U}{\partial x\,\partial y}\right)^2\right]\right\} - \Omega^2 U^2 = c^2 \tag{3.3}$$

where $c^2 > 0$ is a constant (Lagrange multiplier) determined in the course of solving the problem.

We note that when $m = 1$, the condition of optimality does not depend explicitly in the function h.

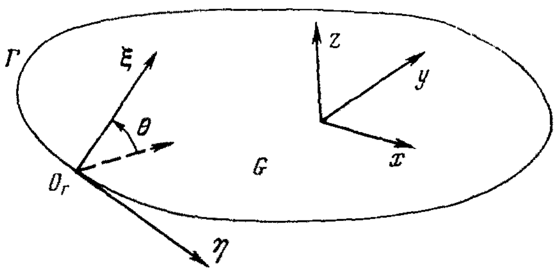

Fig. 1

4. Let us investigate the behavior of the functions h, U and Φ near the contour Γ for the optimal plate in the case of $m = 3$. To do this we introduce a local cylindrical $\eta\xi\theta$-coordinate system at any point O^Γ of the contour Γ, with the η-axis tangent to the contour Γ at the point O_Γ (see Fig. 1). The ξ-axis lies in the plane parpendicular to the η-axis, and the angle θ is counted from the xy-plane in the direction of the ξ-axis. Then for $\theta = 0$ the $\xi\eta$ coordinates represent a Cartesian system in the xy-plane. The condition of optimality (3.3) and the second boundary condition (1.5) in the neighborhood of point O_Γ with $m = 3$ and $\theta = 0$, will now assume the following form:

$$3h^2 \left(\frac{\partial^2 U}{\partial \xi^2}\right)^2 - \Omega_0{}^2 U^2 = c^2, \quad h^3 \frac{\partial^2 U}{\partial \xi^2} = 0 \tag{4.1}$$

which, together with the first boundary condition of (1.5), yields

$$(h)_{\xi=0} = 0, \quad (\partial^2 U / \partial \xi^2)^{-2}_{\xi=0} = 0 \tag{4.2}$$

The above boundary conditions are characteristic for the problems of optimizing edge-hinged beams and plates.

Using the optimality condition (4.1) and the boundary conditions for U, we can write Eq. (1.4) for small ξ in the form

$$\frac{\partial^2}{\partial \xi^2} \left(\frac{\partial^2 U}{\partial \xi^2}\right)^{-2} - \Omega_0{}^2 \left[cU \left(\sqrt{3} \frac{\partial^2 U}{\partial \xi^2}\right)^{-1} + \alpha (\Phi^+ - \Phi^-)\right] = 0 \tag{4.3}$$

Taking into account the boundary condition (4.2) and the condition that the function U vanishes at the boundary Γ, we obtain the following asymptotic representation:

$$U = a_1 \xi + a_2 \xi^2 + \ldots + \xi^\mu (b_0 + b_1 \xi + \ldots) \tag{4.4}$$

The parameter μ must satisfy the conditions $0 < \mu < 2$, $\mu \neq 1$ which follow from (4.2) and the first boundary condition of (1.5). Let us investigate the asymptotic behavior of the function Φ. The Laplace equation for Φ near the point O_Γ can be written in the following asymptotic form:

$$\frac{1}{\xi}\left[\frac{\partial}{\partial \xi}\left(\xi \frac{\partial \Phi}{\partial \xi}\right) + \frac{\partial}{\partial \theta}\left(\frac{1}{\xi} \frac{\partial \Phi}{\partial \theta}\right)\right] = 0$$

We seek a solution of this equation in the form $\Phi = g(\theta)\, \xi^\varkappa + o(\xi^\varkappa)$, where $\varkappa$ is a parameter to be determined and g is a function of the angle θ. Substituting this expression into the Laplace equation and disregarding the infinitesimals of the higher order in ξ, we obtain

$$g'' + \varkappa^2 g = 0$$

We note that since the problem for Φ is antisymmetric with respect to the xy-plane, the function Φ must be odd in z and consequently the function g must satisfy the condition that $g(\pi) = 0$.

The boundary condition for Φ readily yields the boundary condition for g

$$\frac{\partial \Phi}{\partial z} = \frac{1}{\xi}\left(\frac{\partial \Phi}{\partial \theta}\right)_{\theta=0} = \xi^{\varkappa-1} (g')_{\theta=0} = \tag{4.5}$$
$$a_1 \xi + a_2 \xi^2 + \ldots + \xi^\mu (b_0 + b_1 \xi + \ldots)$$

We determine the parameters $\varkappa$ and μ in the usual manner, namely by inspecting successively various versions of the relations connecting these parameters. Without going into details, we formulate the final result. The relations (4.3) — (4.5) determine the following unique values for $\varkappa$ and μ: $\mu = {}^3/_2$, $\varkappa = 2$.

Substituting the asymptotics obtained into the condition of optimality (4.1), we obtain the following asymptotic expression for h:

$$h = \frac{c}{\sqrt{3}} \sqrt{\xi} + \ldots \tag{4.6}$$

The asymptotics (4.6) and (4.4) with $\mu = {}^3/_2$ has the same form as that for the optimal plates oscillating in vacuum [3].

5. When h is fixed, the fundamental frequency Ω_0 depends on the parameter α. It can be shown that Ω_0 is a monotonously decreasing function of the parameter α. Below we consider the case of optimal plates.

To each value of the parameter α there corresponds an optimal distribution of thickness h_* and a certain optimal value of the fundamental frequency Ω_{0*} which is a decreasing function of α.

In fact, let $(\Omega_{0*}{}^2, U, h)$ represent the optimal solution for some value of α and let $(\Omega_{0*}{}^2 + d\Omega_{0*}{}^2, U + \delta U, h + \delta h)$ be the optimal solution for the value of parameter $(\alpha + d\alpha)$. Then from the condition (3.2) follows

$$d\Omega_{0_*}^2 [\alpha I_2(U) + I_3(h, U)] + d\alpha [\Omega_{0_*}^2 I_2(U)] +$$
$$\iint_G [L_1(h, U) - \Omega_{0_*}^2 (hU + \alpha L_2 U)] \delta U dxdy +$$
$$\iint_G \Big\{ mh^{m-1} \Big[\Big(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \Big)^2 - 2(1-\nu) \Big(\frac{\partial^2 U}{\partial x^2} \frac{\partial^2 U}{\partial y^2} - \Big(\frac{\partial^2 U}{\partial x \partial y} \Big)^2 \Big) \Big] -$$
$$\Omega^2 U^2 \Big\} \delta h dxdy = 0$$

The functions h and $h + \delta h$ must satisfy the isoperimetric condition (3.1), therefore we have

$$\iint_G \delta h dxdy = 0$$

and since the functions U and h satisfy the condition of optimality (3.3) and Eq. (1.4), then

$$\frac{d\Omega_{0*}^2}{d\alpha} = -\Omega_{0*}^2 \frac{I_2(U)}{\alpha I_2(U) + I_3(h, U)}$$

We showed before that the operator L_2 is positive. This implies that the integrals $I_2(U)$ and $I_3(h, U)$ are positive, therefore $d\Omega_{0*}{}^2/d\alpha < 0$ and the function $\Omega_{0*}{}^2$ is a strictly decreasing function of α.

6. Consider the case when the bending stiffness D is linearly dependent on the control function h. This case corresponds to a three-layer plate. By h we will understand the thickness of the outer load-carrying layers. Let us set $\lambda = \alpha\Omega^2$. Then the boundary eigenvalue problem will assume the form

$$L_1(h, U) = \lambda (hU / \alpha + L_2 U)$$

We consider the plane problem for a long rectangular plate hinged along the long edges which are parallel to the y-axis. We assume that the plate thickness does not vary in the y-direction, i.e. $h = h(x)$, and we shall carry out investigations in the xz-plane, since derivatives with respect to y are equal zero. Here we shall assume fixed not the volume of the plate, but the area of transverse cross section with the plane perpendicular to the y-axis. The equations describing the oscillations of the plate will be, in this case,

$$\frac{d^2}{dx^2}\Big(h \frac{d^2 U}{dx^2} \Big) = \lambda \Big(\frac{hU}{\alpha} + L_2 U \Big) \qquad (6.1)$$

It was shown in [1] that the expression for L_2 has the form

$$L_2U = \int_{-1}^{1} K(t,x)\,U(t)\,dt, \quad K(t,x) = \frac{1}{\pi}\ln\left[\frac{1+r(t,x)}{1-r(t,x)}\right]$$

$$r(t,x) = \left[\frac{(1-x)(1+t)}{(1-t)(1+x)}\right]^{1/2}$$

The boundary and optimality conditions now become

$$U(-1) = U(1) = 0, \quad \left(h\frac{d^2U}{dx^2}\right)_{x=-1} = \left(h\frac{d^2U}{dx^2}\right)_{x=1} = 0. \tag{6.2}$$

$$\left(\frac{d^2U}{dx^2}\right)^2 - \frac{\lambda}{\alpha}U^2 = c^2 \tag{6.3}$$

Let us consider the limiting case when $\alpha \to \infty$ which corresponds either to thin plates, or to a fluid of infinite density. The equation (6.1) and the optimailty condition (6.3) will now become

$$\frac{d^2}{dx^2}\left(h\frac{d^2U}{dx^2}\right) = \lambda\int_{-1}^{1} K(t,x)\,U(t)\,dt \tag{6.4}$$

$$(d^2U/dx^2)^2 = c^2 \tag{6.5}$$

Taking into account the symmetry of the problem with respect to the point $x = 0$, we shall seek the solution in the interval $[-1,0]$ after specifying the following boundary conditions at the point $x = 0$:

$$\left(\frac{dU}{dx}\right)_{x=0} = 0, \quad \frac{d}{dx}\left(h\frac{d^2U}{dx^2}\right)_{x=0} = 0 \tag{6.6}$$

This condition implies the absence of a shearing force at the center. We also note that the function $h(x)$ does not vanish within the interval $[-1,1]$. In fact, let $h(x_1) = 0$ and let us choose U in the form of a piecewise linear function

$$U = \begin{cases} (1+x)(1-x_1), & -1 \leqslant x \leqslant x_1 \\ (1+x_1)(1-x), & x_1 \leqslant x \leqslant 1 \end{cases}$$

The above function satisfies the boundary conditions, and the Rayleigh relation is equal to zero, irrespective of the choice of h; consequently such a function h is not optimal.

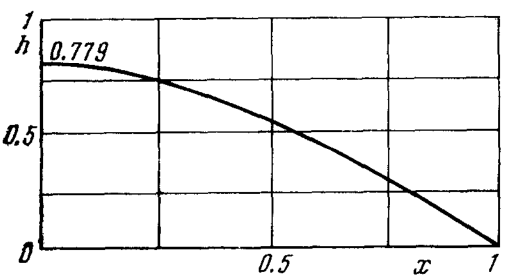

Fig. 2

We have already said that when D is linearly dependent on h, the condition of optimality does not contain h. Let us solve Eq. (6.5) for U with the boundary conditions (6.2). We have

$$U = {}^1/_2 c\,(x^2 - 1)$$

Substituting this expression into (6.4), we obtain

$$\frac{d^2}{dx^2}h = \frac{\lambda}{2}\int_{-1}^{1} K(t,x)(t^2-1)\,dt$$

The boundary conditions of (6.2) and (6.6) are transformed into the boundary conditions for h, namely $h(1) = dh(0)/dx = 0$. The solution of the boundary value problem for h has the form

$$h = \frac{\lambda}{2} g(x), \quad g(x) \equiv \int_{-1}^{x} \int_{0}^{\eta} \int_{-1}^{1} K(t, \xi)(t^2 - 1)\, dt d\xi d\eta$$

Using the expression obtained for h and the isoperimetric condition, we find that

$$\lambda = \left[\int_0^1 g(x)\, dx\right]^{-1} \approx 2.269$$

Figure 2 shows the optimal thickness distribution (because of symmetry, only the region $x > 0$ is shown).

We shall now show that when D is linearly dependent on h ($\alpha \to \infty$), the necessary condition of optimality also becomes sufficient. In fact, let h^* and U^* be solutions satisfying the condition of optimality, h be an arbitrary thickness distribution and U be the corresponding deflection function. Then

$$\Delta\lambda = \lambda(h^*) - \lambda(h) = \frac{I_1(h^*, U^*)}{I_2(U^*)} - \frac{I_1(h, U)}{I_2(U)}$$

We shall show that $\Delta\lambda \geqslant 0$. To do this, we take into account the condition of optimality (3.3), the isoperimetric condition (3.1) and the properties of the functions h^*, U^* and h, U, to arrive at the following estimates:

$$\Delta\lambda = \frac{I_1(h^*, U^*)}{I_2(U^*)} - \frac{I_1(h, U)}{I_2(U)} \geqslant \frac{I_1(h^*, U^*)}{I_1(U^*)} - \frac{I_1(h, U^*)}{I_2(U^*)} =$$

$$\frac{1}{I_2(U^*)} \iint_G \left\{\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right)^2 - 2(1 - \nu)\left[\frac{\partial^2 U}{\partial x^2}\frac{\partial^2 U}{\partial y^2} - \right.\right.$$

$$\left.\left.\left(\frac{\partial^2 U}{\partial x \partial y}\right)^2\right]\right\}(h^* - h)\, dxdy = \frac{1}{I_2(U^*)} \iint_G c^2 (h^* - h)\, dxdy = 0$$

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STRESS FUNCTIONS AND SOME A PRIORI ESTIMATES IN PLATE BENDING THEORY

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A system of equations of the Reissner plate bending theory is formulated in terms of stress functions. An estimate of the elastic energy is deduced from the variational principle for the stress functions. By using this estimate it is proved that